SOL 6.6 – Operations with Fractions

6.6 The student will

a) multiply and divide fractions and mixed numbers; and

b) estimate solutions and then solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions.

Understanding the Standard:

- Simplifying fractions to simplest form assists with uniformity of answers.
- Addition and subtraction are inverse operations as are multiplication and division.
- It is helpful to use estimation to develop computational strategies. For example,
- $2\frac{7}{8} \cdot \frac{3}{4}$ is about $\frac{3}{4}$ of 3, so the answer is between 2 and 3.
- When multiplying a whole by a fraction such as $3 \times \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole.
- When multiplying a fraction by a fraction such as $\frac{2}{3} \times \frac{3}{4}$, we are asking for part of a part.
- When multiplying a fraction by a whole number such as $\frac{1}{2} \times 6$, we are trying to find a part of the whole.

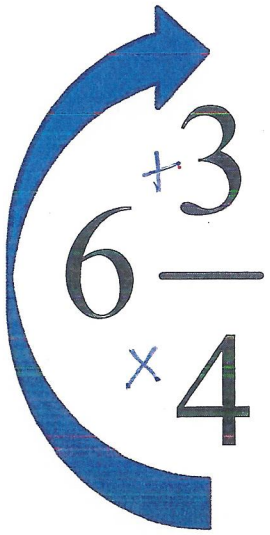
Review:

- A **proper fraction** is a fraction where the numerator is smaller than the denominator, which indicates it is between 0 and 1 whole.
- An **improper fraction** is a fraction where the numerator is larger than the denominator, which indicates it is larger than 1 whole.
- A **mixed number** is a whole number and a proper fraction, indicating that it is larger than 1 whole.
- Because **improper fractions** and **mixed numbers** both represent numbers larger than one whole they are interchangeable.

SOL6.6 – Changing between Improper Fractions and Mixed #s

Changing MIXED Numbers to IMPROPER fractions

- Multiply the denominator and the whole number
- Add the product and the numerator
- The denominator stays the same



$\frac{27}{4}$

Changing IMPROPER fractions to MIXED Numbers

- Divide
- The quotient is the whole number
- The remainder is the numerator
- The denominator stays the same

$\frac{12}{5}$ $2\frac{2}{5}$

$$5\overline{)12}\quad 2$$
$$-10$$
$$2$$

SOL 6.6 – Greatest Common Factor and Least Common Multiple

Greatest Common Factor – GCF: Used to reduce fractions

The meaning of factors:	Finding the GCF of…	Use the GCF to Simplify or Reduce a Fraction
Factor → 3 × Factor → × 4 Product → 12	16: 1×16, 2×8, 4×4 48: 1×48, 2×24, 3×16, 4×12, 6×8	$\frac{16 \div 16}{48 \div 16} = \frac{1}{3}$ $\frac{16}{16} = 1$ thus you are ÷ by 1

Least Common Multiple – LCM: Counting by a

- Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27,…
- Multiples of 9: 9, 18, 27, 36, 45, 54,…
- Use the **LCM to find common denominators** to compare fractions or add fractions

Compare (<,>,=)	Add or Subtract
$\frac{2}{3} \_\_ \frac{1}{9}$	$\frac{2}{3} + \frac{1}{9}$
$\frac{2 \times 3}{3 \times 3} = \frac{6}{9}$ & $\frac{1}{9} = \frac{1}{9}$	$\frac{2 \times 3}{3 \times 3} = \frac{6}{9} + \frac{1}{9} = \frac{1}{9}$
$\frac{6}{9} > \frac{1}{9}$	$\frac{6}{9} + \frac{1}{9} = \frac{7}{9}$

SOL 6.6 – Adding Fractions

Find the common denominator

- Use the LCM. Remember, what ever you do to the numerator you must do to the denominator.

$2\frac{1}{2} \times 4 \times 4 \quad 2\frac{4}{8}$

$+3\frac{5}{8} \quad 3\frac{5}{8}$

Add

- Add the whole number and the numerator
- Denominator stays the same.

$5\frac{9}{8}$

Simplify

- If mixed and improper, REGROUP.
- If proper, REDUCE using the GCF.
- If imprpoper, Divide to get a mixed number.

$5 + \frac{9}{8}$

$5 + 1\frac{1}{8} = 6\frac{1}{8}$

$\frac{9}{8} \rightarrow 1\frac{1}{8}$

$8\overline{)9}$

-8

1

SOL 6.6 – Subtracting Fractions

Find the common denominator

- Use the LCM. Remember, what ever you do to the numerator you must do to the denominator.

$6\frac{1}{2} \times 3 \quad 6\frac{3}{6}$

$-3\frac{2}{3} \times 2 \quad 3\frac{4}{6}$

Can you subtract?

- If no, borrow 1 from the whole number.
- Rewrite the 1 as a fraction = 1.
- If there is already a fraction on the mixed number, add the two fractions.

$5 + \frac{6}{6}$

$6\frac{3}{6} = 5\frac{9}{6}$

$-3\frac{4}{6} = 3\frac{4}{6}$

Subtract

- Subtract the whole number and the numerator
- Denominator stays the same.

$2\frac{5}{6}$

Simplify

- If mixed and improper, REGROUP.
- If proper, REDUCE using the GCF.
- If imprpoper, Divide to get a mixed number.

$2\frac{5}{6}$

SOL 6.6 – Multiplying Fractions

Change the whole #s and mixed #s to improper fractions

- Refer to 4 pages ahead.

$\frac{4}{1} \times \left(\frac{2\times10+1}{2\times10}\right)$

$\frac{4}{1} \times \frac{21}{10}$

Cross cancel

- Basically this is reducing inside the problem.

$\frac{2\cancel{4}}{1} \times \frac{21}{\cancel{10}_{5}} =$

Muitiply

- Multipy the numerators and then the denominators.

$\frac{42}{5}$

Simplify

- If proper, REDUCE using the GCF.
- If imprpoper, Divide to get a mixed number.

$5\overline{)42}$ → 8, -40, 2

$8\frac{2}{5}$

SOL 6.6 – Dividing Fractions

Change the whole #s and mixed #s to improper fractions

- Refer to 5 pages ahead.

$3\frac{1}{3} \div \frac{5}{1}$

$\frac{10}{3} \div \frac{5}{1}$

Change × to ÷ and use the reciprocal of the 2nd fraction

flip

$\frac{10}{3} \times \frac{1}{5}$

Cross cancel

- Basically this is reducing inside the problem.

$\frac{2\cancel{10}}{3} \times \frac{1}{\cancel{5}_{1}}$

Muitiply

- Multipy the numerators and then the denominators.

$\frac{2}{3}$

Simplify

- If proper, REDUCE using the GCF.
- If imprpoper, Divide to get a mixed number.

$\frac{2}{3}$

Essential Understandings:

How are multiplication and division of fractions and multiplication and division of whole numbers alike?

Fraction computations can be approached in the same way as whole number computation, applying those concepts to fractional parts

What is the role of estimation in solving problems?

Estimation helps determine the reasonableness of answers.

Essential Knowledge & Skills:

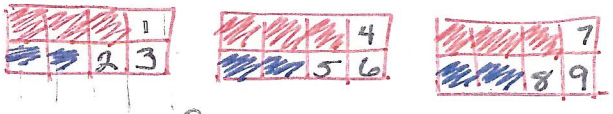
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Multiply and divide with fractions and mixed numbers. Answers are expressed in simplest form.
- Solve single-step and multistep practical problems that involve addition and subtraction with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form.
- Solve single-step and multistep practical problems that involve multiplication and division with fractions and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form.

Practice:

1. Jill had 3 full pans of brownies.
   - Each pan was the same size.
   - She gave $\frac{3}{8}$ of the original amount of brownies to her friends.
   - She gave $\frac{1}{4}$ (×2 → $\frac{2}{8}$) of the original amount of brownies to her teacher.

   Exactly how many pans of brownies does Jill have left over?



$\frac{9}{8} = 1\frac{1}{8}$

2. Kendra recorded the amount of water she used in one week for four activities.
   a. What is the total amount of water, in gallons, recorded for these activities?
   b. How much more water was used doing laundry than cooking?

Activity	Amount of Water (in gallons)
Bathing	$50\frac{2}{3}$
Doing Laundry	$33\frac{1}{4}$
Washing Car	$8\frac{3}{8}$
Cooking	$10\frac{3}{4}$

(a) $50\frac{2}{3} \times 8$, $33\frac{1}{4}$, $10\frac{3}{4}$ → 1 whole

$+ \quad 8\frac{3}{8} \times 3$

102

$\frac{16}{24}$

$\frac{9}{24}$

$102 + \frac{25}{24} = 1\frac{1}{24}$

(a) $103\frac{1}{24}$

$32\frac{4}{4}$

$33\frac{1}{4} = 32\frac{5}{4}$

$-10\frac{3}{4} \quad -10\frac{3}{4}$

$22\frac{2}{4} =$

(b) $22\frac{1}{2}$

3. What is the product of $1\frac{1}{3}$ and $\frac{7}{10}$ in simplest form?

$1\frac{1}{3} \times \frac{7}{10}$

$\frac{4}{3} \times \frac{7}{10_5} = \frac{14}{15}$

4. The length of a rope is $21\frac{3}{8}$ feet. James cut $2\frac{3}{4}$ feet from this length of rope to use on a project. Exactly what length of rope remained unused?

$21\frac{3}{8}$ $\quad 20\frac{11}{8}$

$2\frac{3}{4}$ $\quad 2\frac{6}{8}$ $\quad -2\frac{6}{8}$

$18\frac{5}{8}$

5. Each $\frac{1}{2}$ cup of milk has 4 grams of protein. Exactly how many grams of protein are in $3\frac{1}{2}$ cups of milk?

a. 7 grams
b. 8 grams
c. 14 grams
d. 28 grams